differing materially from the rest of the series. The purple alloy of gold and aluminium M, $AuAl_2$, is almost certainly a true chemical compound, the solidified mass being as nearly uniform in composition as may be. The uniformity of the alloy (J) of platinum with 10 per cent. of rhodium is of much interest, in view of the important part which the alloy is playing in pyrometric work.

Conducting the experiments, the results of which are embodied in the present paper, has been very laborious, and although, as already stated, no complete series of the alloys of any two metals has been examined, quite sufficient data have been collected to afford valuable guidance to the metallurgist, who will now know what behaviour may be expected from the other members of the groups of the alloys in question. The gold-platinum series of alloys are of industrial importance, as native gold is so often associated with platinum, and it is somewhat surprising to find that assays made on pieces of metal cut from the exterior of an ingot cannot be trusted to represent the composition of the mass. The aim of the investigation has been to show, that notwithstanding the great difficulty which attends the preparation of alloys of metals with very high melting points, it is possible to elicit from them the same kind of information which has proved to be so useful in the case of the more ordinary and tractable alloys.

IV. "The Potential of an Anchor Ring." By F. W. Dyson, Fellow of Trinity College, Cambridge. Communicated by Professor J. J. Thomson, F.R.S. Received March 19, 1892.

(Abstract.)

If r, θ, ϕ be the coordinates of any point outside an anchor ring, whose central circle is of radius c, then

$$\int_0^\pi \frac{d\phi}{\sqrt{(r^2+c^2-2cr\sin\theta\cos\phi)}}$$

and

$$\cos\phi\int_0^\pi \frac{\cos\phi\, d\phi}{\sqrt{(r^2+c^2-2cr\sin\theta\cos\phi)}}$$

are solutions of Laplace's equation, which are finite at all external points and vanish at infinity.

Let these be called I and J.

Then $d\mathrm{I}/dz$ and $d\mathrm{J}/dz$ are also solutions of Laplace's equation.

Four sets of solutions are obtained by differentiating these integrals any number of times with respect to c.

Thus, I, $d\mathrm{I}/dc$, $d^2\mathrm{I}/dc^2$, &c., are all solutions of Laplace's equation, finite at all external points and vanishing at infinity.

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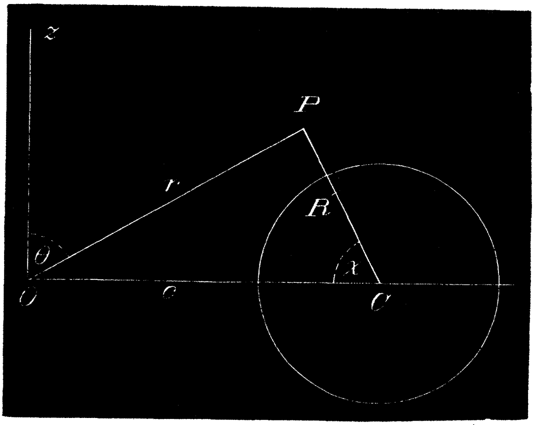
This fact is applied to find the potential of an anchor ring in external space, the velocity potential of a ring moving in any manner in an infinite fluid, the potential of a charged conductor in the form of an anchor ring, and the annular form of rotating fluid.

FIG. 1.



2. Let a section be taken through the axis of the ring, cutting the central circle at C. Let P be a point in this section, and let

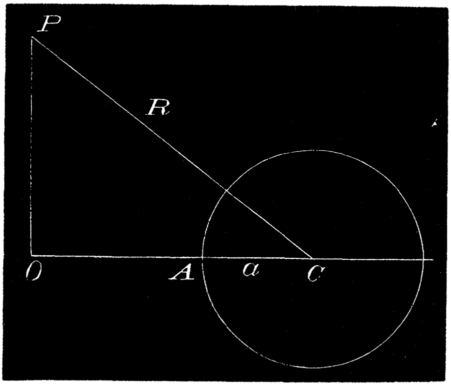
$$\mathrm{CP} = \mathrm{R}, \qquad \mathrm{OC} = c, \qquad \text{and } \mathrm{OCP} = \chi.$$

When $\mathrm{R} < c$, the integrals

$$\int_0^\pi \frac{d\phi}{\sqrt{(r^2+c^2-2cr\sin\theta\cos\phi)}} \quad \text{and} \quad \int_0^\pi \frac{\cos\phi\, d\phi}{\sqrt{(r^2+c^2-2cr\sin\theta\cos\phi)}}$$

are expanded in ascending powers of R/c; the expansions of the integrals $d\mathrm{I}/dc$, $d^2\mathrm{I}/dc^2$, &c., are deduced from these.

FIG. 2.



In this way the values of the integrals at the surface of the ring are found and the required boundary conditions satisfied.

3. The potential of an anchor ring at any point on its axis is shown to be

$$\frac{2\mathrm{M}}{\pi}\int_0^{\pi}\frac{\sin^2\psi\,d\psi}{\sqrt{(\mathrm{R}^2-2a\mathrm{R}\cos\psi+a^2)}},$$

where $\mathrm{R} = \mathrm{CP}$ and $a = \mathrm{CA}$.

This is reduced to elliptic functions and expanded in ascending powers of a/R, giving

$$\mathrm{V} = \mathrm{M}\left\{\frac{1}{\mathrm{R}}-\frac{1}{8}\frac{a^2}{\mathrm{R}^2}-\frac{1}{64}\frac{a^4}{\mathrm{R}^5}-\text{\&c.}\right.$$

$$\left.-2\,\frac{1^2.3^2\ldots\ldots(2n-3)^2(2n-1)}{2^2.4^2\ldots\ldots(2n-2)^2(2n)^2(2n+2)}\frac{a^{2n}}{\mathrm{R}^{2n+1}}-\text{\&c.}\right\}$$

from which it is deduced that at any external point r,θ

$$\mathrm{V} = \frac{\mathrm{M}}{\pi}\left\{\int_0^{\pi}\frac{d\phi}{\sqrt{(r^2+c^2-2cr\sin\theta\cos\phi)}}\right.$$

$$+\frac{a^2}{8}\frac{d}{c\,dc}\int_0^{\pi}\frac{d\phi}{\sqrt{(r^2+c^2-2cr\sin\theta\cos\phi)}}-\text{\&c.}$$

$$\left.+(-1)^{n+1}\frac{2a^{2n}}{2n+2}\,\frac{1.3.5\ldots\ldots(2n-3)}{2^2.4^2.6^2\ldots\ldots(2n)^2}\left(\frac{d}{c\,dc}\right)^n\int_0^{\pi}\frac{d\phi}{\sqrt{(r^2+c^2-2cr\sin\theta\cos\phi)}}+\text{\&c.}\right\}$$

This series of integrals is very convergent; the first three are reduced to elliptic functions, and the equipotential surfaces are drawn for the ratios $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, 1, of a/c.

4. The potential of a conductor in the form of an anchor ring is shown to be of the form

$$\mathrm{A}_0\mathrm{I}+\mathrm{A}_1a^2\frac{d\mathrm{I}}{c\,dc}+\mathrm{A}_2a^4\left(\frac{d}{c\,dc}\right)^2\mathrm{I}+\text{\&c.}$$

A_0, A_1, A_2, &c., are determined, and it is shown that on the surface of the ring

$$\mathrm{V} = \frac{\mathrm{A}_0}{c}\left\{\lambda_0+2-\frac{4\lambda_0+8\lambda_0+5}{2^4}\sigma^2+\frac{192\lambda_0+416\lambda_0^{\ 2}+448\lambda_0+179}{2^{11}}\sigma^4+\ldots\right\}$$

and

$$\frac{d\mathrm{V}}{dn} = \frac{\mathrm{A}_0}{ac}\left\{-1+\left(\frac{2\lambda_0+1}{2}-\frac{24\lambda_0^{\ 2}+7}{2^6}\sigma^2\right)\left(\frac{\sigma^2}{2}+\sigma\cos\chi\right)\right.$$

$$+\left(\frac{8\lambda_0+1}{2^4}-\frac{36\lambda_0^{\ 2}+\lambda_0+13}{2^7}\sigma^2\right)\sigma^2\cos 2\chi$$

$$\left.+\frac{16\lambda_0-3}{2^6}\sigma^3\cos 3\chi+\frac{11(8\lambda_0-1)}{2^{10}}\sigma^4\cos 4\chi+\text{\&c.}\right\},$$

and the

$$\text{charge} = \pi A_0, \text{ where } \sigma \text{ denotes } a/c \text{ and } \lambda_0 = \log\frac{8c}{a} - 2.$$

5. The velocity potential of a ring moving in an infinite fluid is next found—

(1) When the ring moves parallel to its axis;
(2) When the ring moves perpendicularly to its axis;
(3) When the ring rotates about a diameter of its central circle.

The stream line function is found—

(1) For motion parallel to the axis;
(2) For the cyclic motion through the ring.

Thus, in the first case it is shown that

$$\Phi = wa^2\left\{\left[1+\frac{4\lambda_0+1}{8}\delta_0^2+\frac{16\lambda_0^2+104\lambda_0+33}{64}\delta_0^4\right]\frac{dI_1}{dz}-\frac{12\lambda_0+9}{16}\delta_0^4\frac{dI_2}{dz}\right.$$
$$\left.+\left[\frac{\delta_0^4}{16}+\frac{112\lambda_0+91\frac{2}{3}}{256}\delta_0^6\right]\frac{dI_3}{dz}-\frac{\delta_0^6}{48}\frac{dI_4}{dz}+\frac{5\delta_0^8}{1024}\frac{dI_5}{dz}\right\},$$

where $$I_1 = \int_0^\pi \frac{c\,d\phi}{\sqrt{(r^2+c^2)-2cr\sin\theta\cos\phi}},$$

and $$I_n = \left(2c\frac{d}{dc}\right)^{n-1} I_1,$$

and $$\delta_0 = a/2c.$$

The kinetic energy is calculated in each case as far as δ_0^4, the result being that

$$2T = P(u^2+v^2)+Rw^2+A(\omega_1^2+\omega_2^2)+K\kappa^2,$$

where

$$P = \frac{M}{2}\left\{1-\frac{12\lambda_0+1}{4}\delta_0^2-\frac{72\lambda_0^2-24\lambda_0+7}{16}\delta_0^4-\dots\right\},$$

$$R = M\left\{1-\frac{4\lambda_0+1}{4}\delta_0^2+\frac{16\lambda_0^2+8\lambda_0-23}{32}\delta_0^4+\dots\right\},$$

$$A = M\frac{c^2}{2}\left\{1-\frac{12\lambda_0+7}{4}\delta_0^2+\frac{144\lambda_0^2-36\lambda_0+17}{32}\delta_0^4+\dots\right\},$$

$$K = \rho\frac{c}{2}\left\{\lambda_0-\frac{4\lambda_0^2+8\lambda_0+1}{4}\delta_0^2-\frac{64\lambda_0^3+188\lambda_0^2+192\lambda_0-49}{128}\delta_0^4+\dots\right\},$$

where M is the mass of the fluid displaced.

The linear momentum of the cyclic motion is also found to be

$$\pi c^2 \kappa \rho \left\{ 1 - 4(\lambda_0 + 1)\, \delta_0^{\,2} - \frac{4\lambda_0^{\,2} + 5\lambda_0 - 2}{2}\, \delta_0^{\,4}, \text{ \&c.} \right\}.$$

A few cases of motion of the ring are then discussed.

6. The annular form of fluid rotating in relative equilibrium is next considered, when the radius of the mean circle of the ring is at least twice as great as the mean radius of the cross-section.

The equation of the cross-section is assumed to be

$$\rho = a\,(1 + \beta_1 \cos\chi + \beta_2 \cos 2\chi + \ldots\ldots),$$

where β_1, β_2, &c., are small quantities.

Taking the centre of gravity of the cross-section as origin, β_1 is seen to be of the 5th order, and it is shown that, as far as the 4th order,

$$\frac{\omega^2}{\pi} = \left(\lambda_0 + \frac{3}{4}\right)\sigma^2 - \frac{1}{8}\left(\lambda_0 + \frac{19}{12}\right)\sigma^4,$$

$$\beta_2 = \frac{\frac{5}{8}\,\sigma^2 \left\{ \left(\lambda_0 + \frac{7}{12}\right) + \frac{5\,\sigma^2}{48}\left(\lambda_0 - \frac{107}{120}\right) \right\}}{1 - \left(\lambda_0 + \frac{1}{2}\right)\sigma^2},$$

$$\beta_3 = \frac{5}{128}\left(\lambda_0 - \frac{7}{24}\right)\sigma^3,$$

$$\beta_4 = \frac{75\lambda_0^{\,2} + 90\lambda_0 + 23}{256}\,\sigma^4.$$

The shape of the cross-section is roughly elliptical, the major axis of the ellipse being perpendicular to the axis of revolution.

V. "On the Residues of Powers of Numbers for any Composite Modulus, Real or Complex." By GEOFFREY T. BENNETT, B.A. Communicated by Professor CAYLEY, F.R.S. Received April 8, 1892.

(Abstract.)

The present work consists of two parts, with an Appendix to the second. Part I deals with real numbers, Part II with complex.

In the simple cases, when the modulus is a real number which is an odd prime, a power of an odd prime, or double the power of an odd prime, we know that there exist primitive roots of the

FIG. 1.

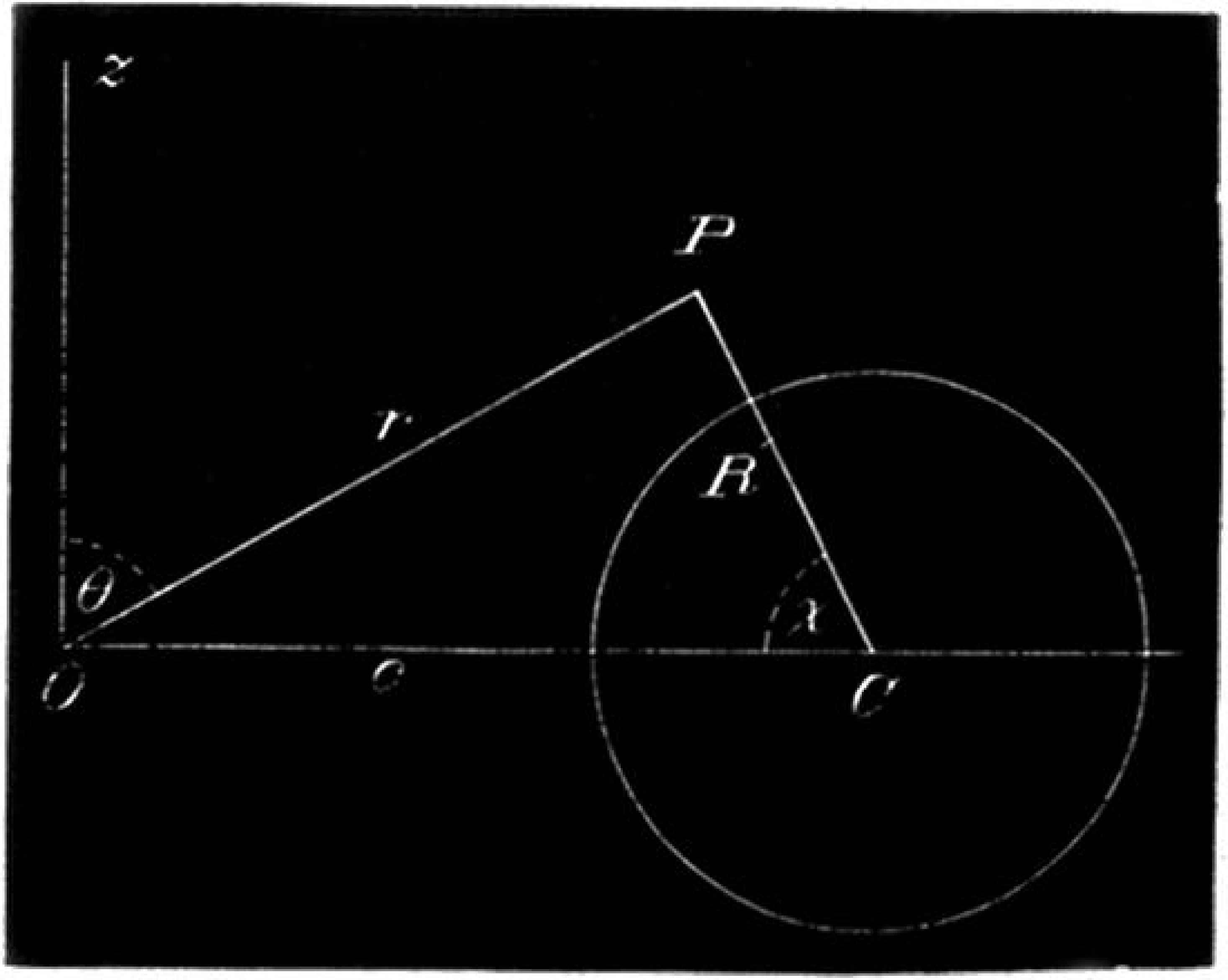


FIG. 2.